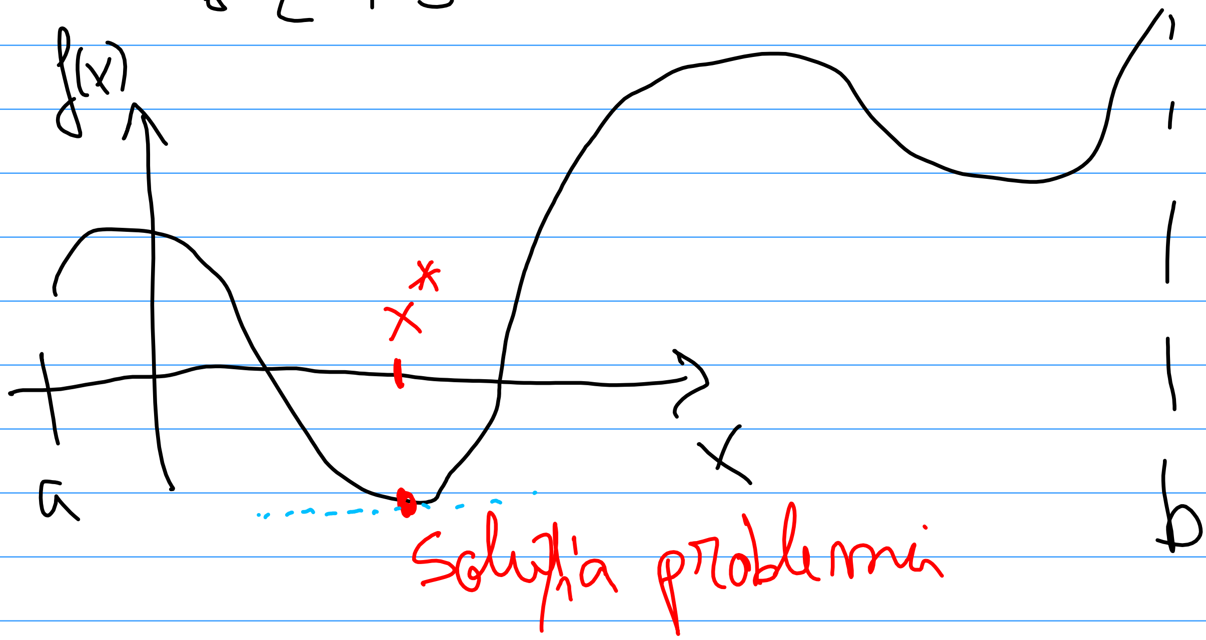
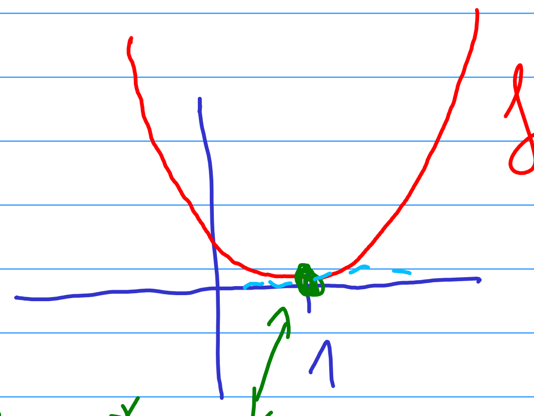
Tehnici de Optimizare

$$f:[a,b]\to\mathbb{R}$$

1D



$$\min_{x\in[a,b]} f(x) \to \text{soluția } x^*$$

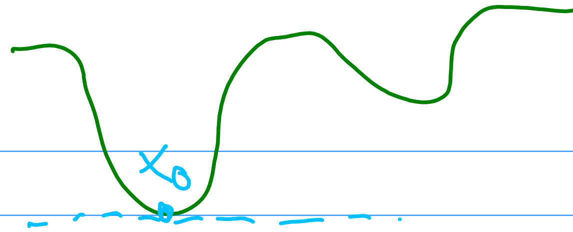


$$f(x)=(x-1)^2 \geq 0$$

$$f(1)=0$$

$\Rightarrow$ Soluția este $x^*=1$

$$f(x)=e^{x^2}+x^2+x+1$$



→ tangenta paralelă cu Ox

→ derivata?

ec. tangentei: $y = \underbrace{f'(x_0)}\cdot(x-x_0)+f(x_0)$

$= f'(x_0)\,x - f'(x_0)x_0 + f(x_0)$

panta dreptei $= f'(x_0)$

dreaptă orizontală are ecuația: $y = \underbrace{0}_{\text{panta zero}}\cdot x + c$

Condiție de optimalitate:

x^* este soluția pt problema

$\min\limits_{x\in[a,b]} f(x)$ și $x^* \in (a,b)$

atunci $f'(x^*) = 0$

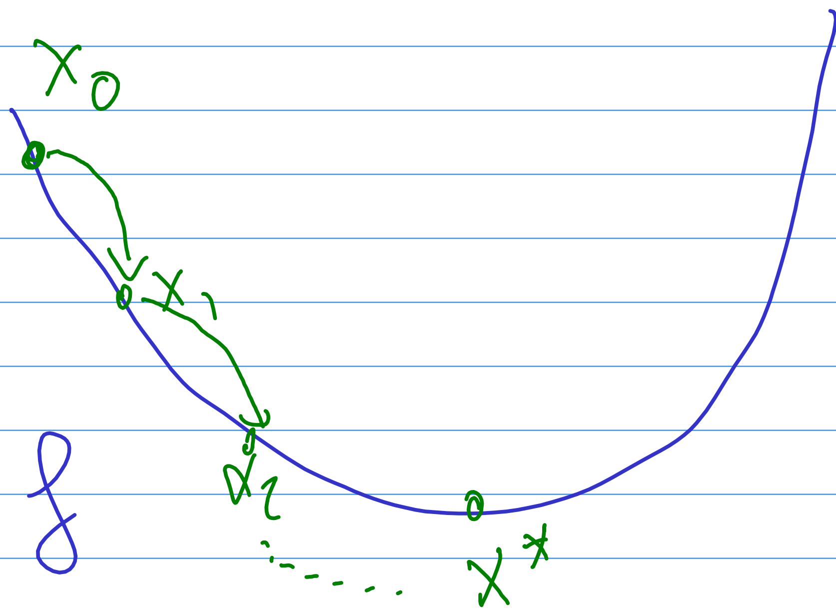
$f(x) = e^{x^2} + x^2 + x + 1$

$f'(x) = 2x\,e^{x^2} + 2x + 1$

Rezolvăm $2xe^{x^2}+2x+1=0$

$x=?$ imposibil analitic (?)

Algo optimizare:



generează un șir $(x_n) \to$ o soluție

1. $x\in\mathbb{R}^n$, $A\in\mathbb{R}^{n\times n}$ sym pos. def, $b\in\mathbb{R}^n$

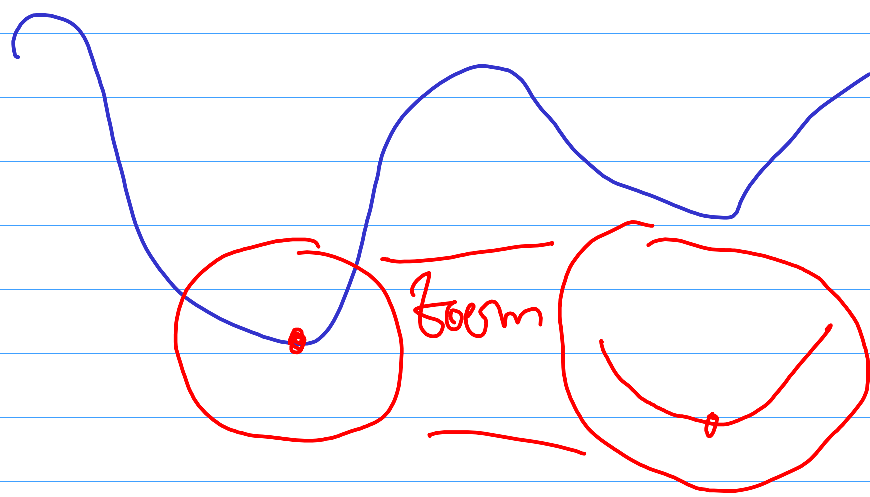
$$f(x)=\frac{1}{2}x^TAx-b^Tx$$

$$x \mapsto \underset{(1\times n)}{(x_1 \dots x_n)}\underset{(n\times n)}{\begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix}}\underset{(n\times 1)}{\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix}} - \underset{(1\times n)}{(b_1 \dots b_n)}\underset{(n\times 1)}{\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix}}\in\mathbb{R}$$

Exemplu: $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $b = 0$

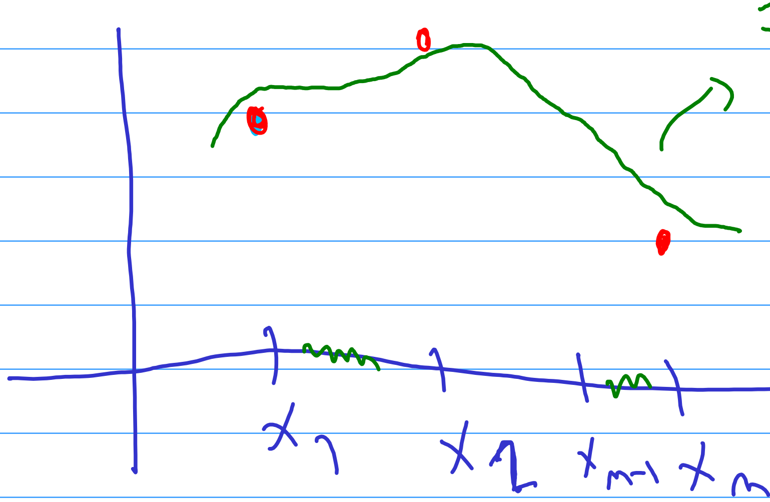
$x \in \mathbb{R}^2$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $f(x) = (x_1, x_2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$= (x_1, x_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1^2 + x_2^2$

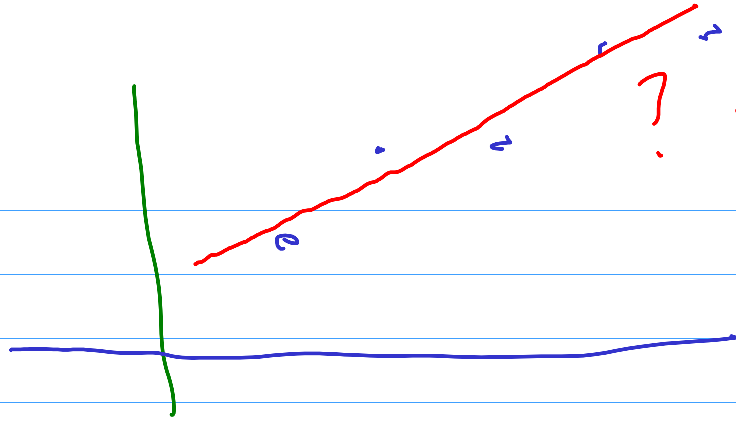


în jurul minimului o funcție regulată seamănă cu o parabolă

(3) Găsirea de modele matematice

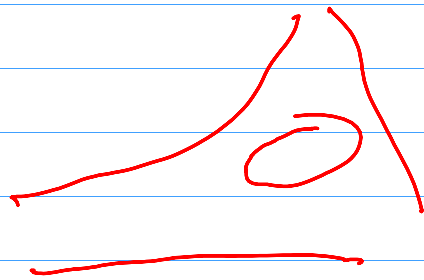


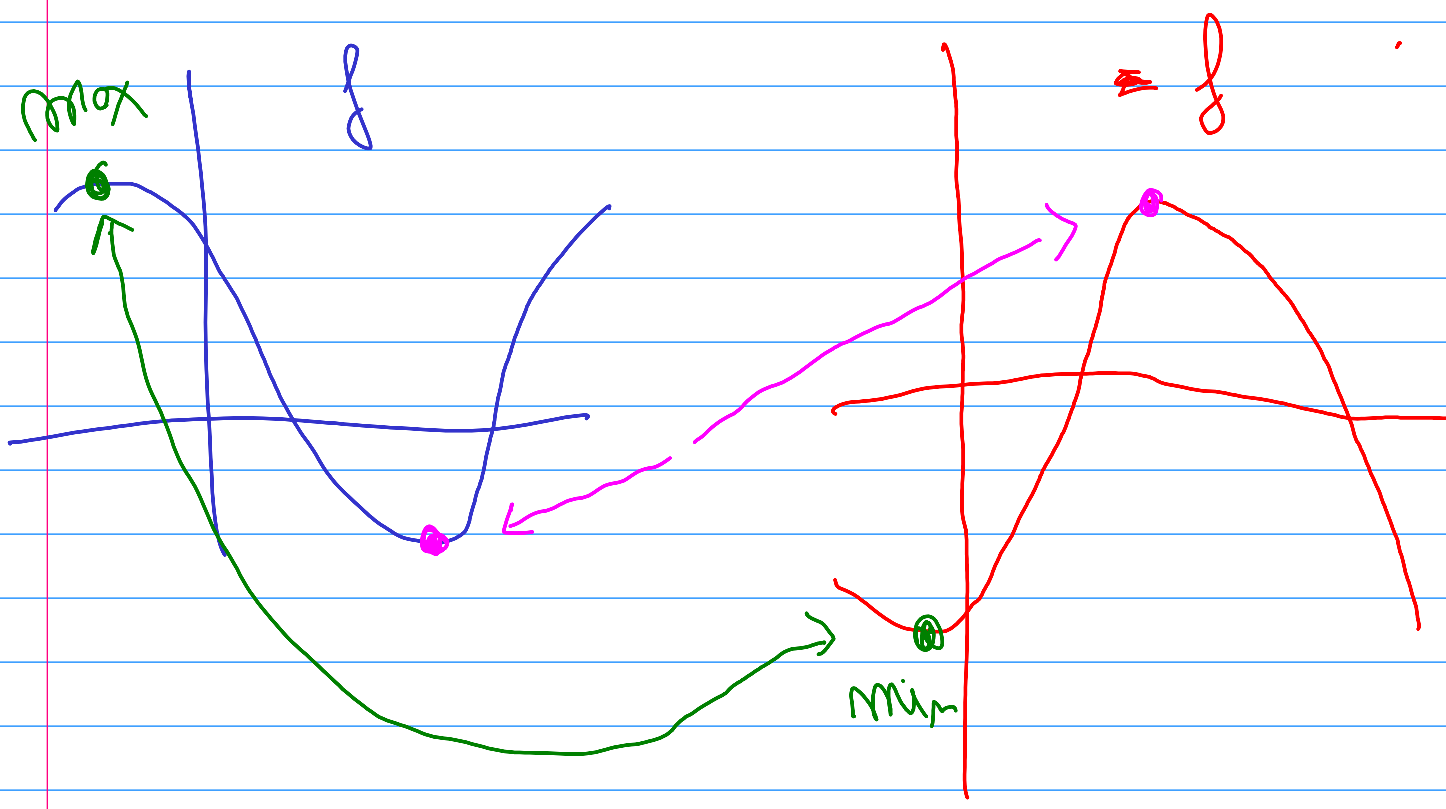
Funcție model care depinde de parametri.



$y = mx + n$

param. necunoscuti





Opt assignment

$A : \{1, 2, 3\} \to \{1, 2, 3\}$

A - bijectivă $\to$ permutări

$$\left.\begin{array}{l} A(1) \to 3 \\ A(2) \to 2 \\ A(1) \to 1 \end{array}\right\} \quad 3 \times 2 \times 1 = 3!$$

$A : \{1, \dots, n\} \to \{1, \dots, n\}$

A bijectivă : $n!$ posibilități

Algo brute force: complexitate $O(n!)$