Numerical Calculation: Problems

1. **Computing Expressions.** Consider the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$.

   a) Test that $\phi^2 - \phi - 1 = 0$.

   b) Compute $\left(\sqrt[6]{\pi^5 + \pi^7}\right)^{4/\sqrt{\phi}}$, show 10 significant digits.

2. **Symbolic Computations.**

   a) Demonstrati folosind calcul simbolic identitatea urmatoare

   $$(a_1^2 + a_2^2)(b_1^2 + b_2^2) = (a_1 b_1 + a_2 b_2)^2 + (a_1 b_2 - a_2 b_1)^2.$$

   b) Factorizati 50! si gasiti numarul de zero-uri al rezultatului.

3. **Derivatives and primitives**

   a) Compute the first and second derivatives of $f(x) = \dfrac{\sin(x^2+1)}{\sqrt{x^4 - x^2 + 1}}$

   b) Compute the primitive of $e^{3x} \sin x$

   c) Compute the limit $\lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n+n^2}\right)^{3n^2}$

4. **Computing definite integral and plotting**

   a) Compute $\int_{-1}^{1} \sqrt{1-x^2}$

   b) Plot the region between the function and the Ox axis.

5. **Solve an equation**

   a) Solve the equation $x^3 - x + 1 = 0$, finding all the roots. (with `polyroots`: write coefficients from the smallest power to the biggest one in a vector, then apply `polyroots`).

   b) Solve the equation $x^3 - x + 1 = 0$ using a solve block starting from the initial condition $x = -1$. What is the root found? Is it among the roots presented at point a)? What happens if we change the initial condition to $x = 1$?

6. **Solve a linear system**

   a) (Using Solve Block)

   $$\begin{cases} 3x + 2y + 5z = 22 \\ 4x - 2y + 3z = 9 \\ x + 5y - z = 8 \end{cases}$$

   b) (Using Matrices: `lsolve(A,b)`) Solve the same system using matrices and vectors.

7. **Linear Programming** Solve the following linear programming problem.

   $$\text{Maximize } (515 \cdot x + 776 \cdot y) \text{ such that } \begin{cases} 13x + 19y \le 40 \cdot 60 \\ 20x + 29y \le 35 \cdot 60 \\ x \ge 10 \\ y \ge 0 \end{cases}$$

Use a **solve block** in Mathcad.

8. **Minimize a non-linear function.** Let $f(x, y, z) = 100(z - y^2)^2 + 100(y - x^2)^2 + (x - 1)^2$.

   a) Find the minimal value of $f(x, y, z)$.

   b) Find the minimal value of $f(x, y, z)$ under the constraint $x + y + z = 1$.

9. **Non-linear minimization** with different outcomes depending on the initialization.

$$f(x, y) = (x^2 - 1)^2(y^2 + 1) + 0.2y^2.$$

   a) Plot 2D contours of the function in the box $[-1.5, 1.5]^2$. Also plot a 3D view of the function. (Use `CreateMesh(f,-1.5,1.5,-1.5,1.5,100,100)`).

   b) Minimize the function starting from the three points:

$$(0, 1.5), (-0.1, 1.5), (0.1, 1.5).$$