Advanced Programming Techniques

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Practical Session #2

Exercise 1. 1. Algorithm A needs $10n^3$ operations to solve a problem. Algorithm B solves the same problem in $1000n^2$ operations. Which algorithm is faster (and for which n)?

2. Algorithm A needs $32n \log_2 n$ operations to solve a problem. Algorithm B solves it in $3n^2$ operations. Which algorithm is faster?

Exercise 2. Consider an algorithm whose execution time for $N = 1000, 2000, 3000$ and 4000 is respectively $5s, 20s, 45s, 80s$. Estimate the execution time for $N = 5000$.

Exercise 3. Let $f(n) = n$. If possible find a function $g(n)$ such that:

- $f(n) \in O(g(n))$ and $f(n) \notin \Omega(g(n))$.
- $f(n) \notin O(g(n))$ and $f(n) \in \Omega(g(n))$.
- $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$.
- $f(n) \notin O(g(n))$ and $f(n) \notin \Omega(g(n))$.

Exercise 4. (a) Show that the execution time for an algorithm is $\Theta(g(n))$ if and only if the worst case is $O(g(n))$ and the best case is $\Omega(g(n))$.

(b) Show that $5n^2 - 3n + 4$ is $\Theta(n^2)$

(c) Show that 2^{n+1} is $\Theta(2^n)$

(d) Discuss why the phrase "*The execution time of algorithm A is at least* $O(n^2)$" does not make sense.

Exercise 5. Class the following by the order of complexity:

$n \log n$	$4/n$	$\sqrt{n}$	2^{2^n}
$\log \log n$	$8n^3$	$8^{\ln n}$	$n/(2+n)$
$\log n^7$	$5^{\ln \log_2 n}$	$(\log_2 n)^3$	$\frac{n}{\log_2(2+n)}$

Exercise 6. Consider A an array of n values sorted in increasing order. We want to search if a value b is present in the array.

(a) Write a pseudo-code for a brute force algorithm searching for b. Analyse the complexity in the best and worst cases.

(b) Propose a dichotomic algorithm for finding b. Analyse its complexity in the best and worst cases.

Exercise 7. Project Euler (`https://projecteuler.net/` contains a collection of computer science problem demanding efficient implementations. The 4th problem is the following:

A palindrome reads the same from right to left or from left to right. The largest palindrome which can be written as the product of two numbers of two digits is $9009 = 91 \times 99$. *Find the largest palindrome which is the product of two numbers of three digits.*

Propose an algorithm to solve the problem and indicate its complexity. Is the algorithm optimal?

Exercise 8. Consider a $N \times N$ matrix of booleans (0 or 1). Propose an algorithm for finding the largest continuous sub-matrix containing only values of 1.

For example the matrix (2D array)

```
1 0 1 1 1 0 0 0
0 0 0 1 0 1 0 0
0 0 1 1 1 0 0 0
0 0 1 1 1 0 1 0
0 0 1 1 1 1 1 1
0 1 0 1 1 1 1 0
0 1 0 1 1 1 1 0
0 0 0 1 1 1 1 0
```

contains a 4×4 sub-matrix containing only ones.